The Official Definition of Potential Energy:

$$\Delta U = -W$$

In English, that means when a conservative force does negative work, we say the potential energy increased. If a conservative force does positive work, there is a loss of potential energy.

Looking at U as a function, we would say

$$U = -\int F\,dx$$

Given F, we can find U. For a simple example, let's say

$F = -kx$ (i.e. Hooke's Law)

$$\therefore \quad U = -\int (-kx)\,dx = \frac{1}{2}kx^2$$

technically, these would be definite integrals → from one position to another.

what if we are given U instead? Can we find F?

of course we can!

$$U = -\int F\,dx$$

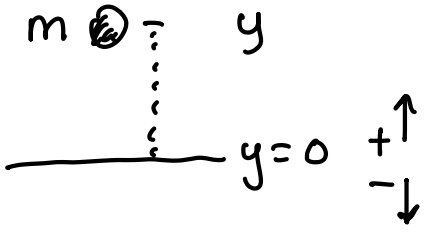
$$dU = -F\,dx$$

← derivatives & integrals are opposites of each other!

$$\frac{dU}{dx} = -F$$

$$F = -\frac{dU}{dx}$$

For example, let's say the potential energy function is given by $U = mgy$ So what is the force?



$$F = -\frac{dU}{dy}$$

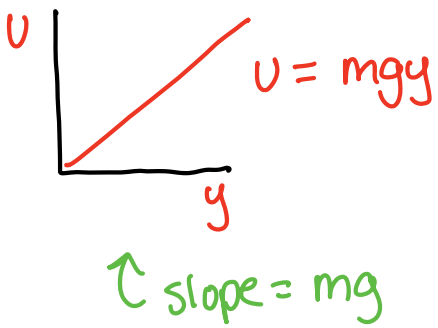
$$F = -\frac{d(mgy)}{dy}$$

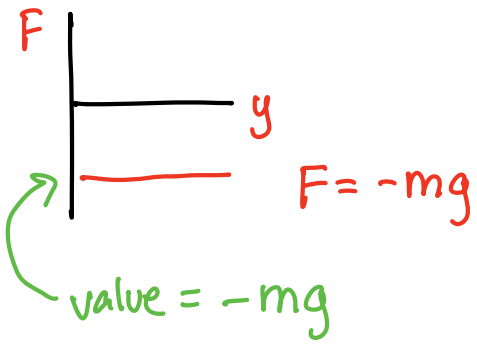
← dy because that is my coordinate!

$$F = -mg$$

Hey! That's just gravity! Negative because its directed down!

we can graph these:





$F = -\frac{dU}{dx}$ means the force is the negative of the slope of the potential energy.

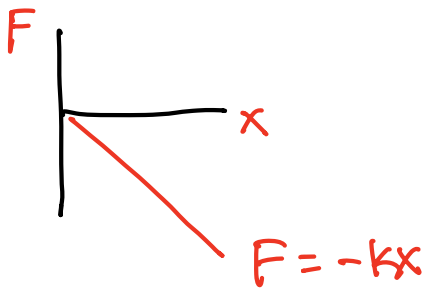
Let's do a spring!

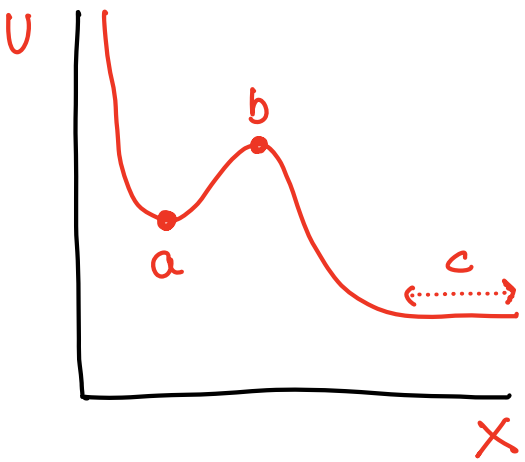
$U = \frac{1}{2}kx^2$

So $F = -\frac{dU}{dx}$

$F = -\frac{d(\frac{1}{2}kx^2)}{dx}$

$F = -kx$





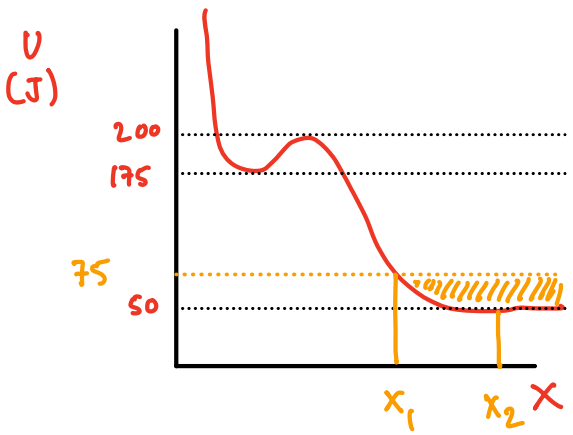
The 3 areas (a, b, c) all have a slope of 0 - so there would be NO force if a particle were there.

EQUILIBRIUM

c - "neutral" Equilibrium [U is constant]

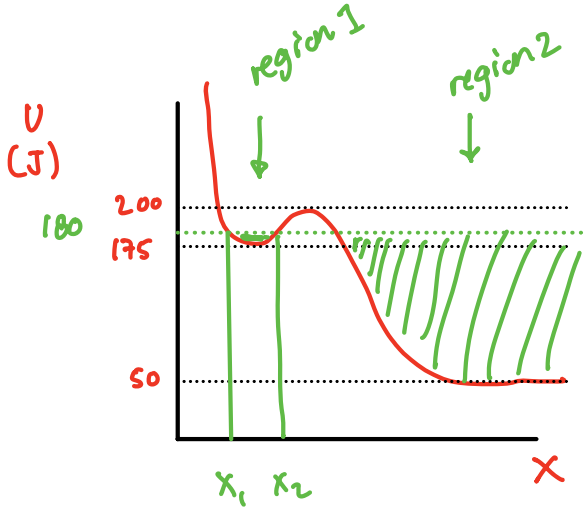
a - "stable" " [U is min]

b - "unstable" " [U is max]

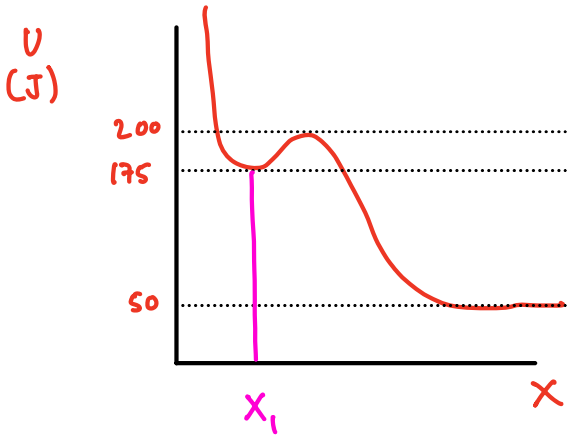


- The graph to the left shows the potential Energy vs. position for some particle.
- Notice the minimum amount of energy the particle could have is 50 J – if it was @ rest in the region of neutral stability.
- If the particle had a total energy of 75J, it could be anywhere in the orange shaded region –
  @ X_1 the energy would be ALL potential
  @ X_2 there would be 50 J of potential and 25 J of kinetic.

Notice how if particle was initially moving to the left – it would slow down, finally stop @ X_1, then go back to the right.



- Now, if the particle instead had a total energy of 180J, there are <u>two</u> regions (shaded green) – on either of the local max @ 200J.
- region 2, the particle would end up moving to the right with K = 130J & U = 50J
- region 1, the particle would oscillate back and forth between the green X_1 & X_2. At those points U = 180 & K = 0. At the minimum, U = 175 & K = 5.



- Now let's imagine a particle is unmoving @ x_1. Its total energy is potential & U = 175. It is at its stable equilibrium point.
- Notice we can "nudge" the particle a little bit, giving it some kinetic energy and it will then oscillate back and forth (like the green example above.
- But what happens if we give the particle an additional 25 J?

"Activation Energy" in Chemistry!

→ It will then be able to reach the max U @ 200 J! Once there, it will end up moving to the right - and then it will lose 150 J of Potential Energy! which means it will gain 150 J of Kinetic energy!

Our nudge of 25 J of energy ended up releasing 150 J! Explosion!